

(i)

Unit-IV
DC & AC Bridges

Resistance:- Resistance is a which oppose the flow of free electrons. It is denoted by 'R' and its units is ohms(Ω)

- these are 3 types these are
- low Resistance [less than 1Ω]
 - medium Resistance [From 1Ω - $0.1M\Omega$]
 - high Resistance [above $0.1M\Omega$]

methods for measuring low & medium & high Resistances:-

~~retometer~~ - ~~ammeter~~ method:-

wheatstone bridge:-

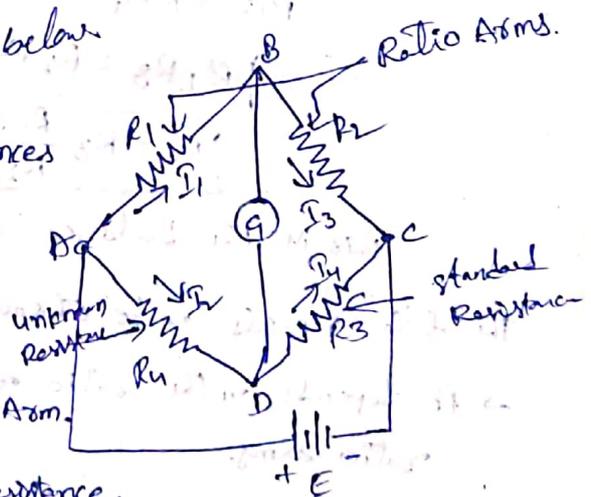
the bridge consists of 4 resistive arms together with a source of e.m.f and a null detector. The galvanometer is used as a null detector. It shows below

the arms consisting the resistances R_1 & R_2 are called Ratio Arms:

the arm consisting the standard known resistance R_3 is called standard Arm.

The resistance R_x is the unknown resistance to be measured.

The battery is connected b/w A & C while the galvanometer is connected b/w B & D.



Balance condition:- when the bridge is balanced the galvanometer carries zero current and it does not show any deflection. Thus bridge works on the principle of null deflection (or) null indication.

To have zero current through galvanometer, the points B & D must be at the same potential. Thus potential across arm AB must be same as the potential across arm AD.

$$\text{Thus } I_1 R_1 = I_2 R_4 \quad \text{--- (1)}$$

As galvanometer current is zero

$$I_1 = I_3 \text{ \& } I_2 = I_4 \quad \text{--- (2)}$$

Considering the battery path under balanced condition.

$$I_1 = I_3 = \frac{E}{R_1 + R_2} \quad \text{--- (3)}$$

$$I_2 = I_4 = \frac{E}{R_3 + R_4} \quad \text{--- (4)}$$

using equations (3) & (4) in equation (1)

$$\frac{E}{R_1 + R_2} \times R_1 = \frac{E}{R_3 + R_4} \times R_4$$

$$R_1 (R_3 + R_4) = R_4 (R_1 + R_2)$$

$$R_1 R_3 + R_1 R_4 = R_1 R_4 + R_2 R_4$$

$$\boxed{R_1 R_3 = R_2 R_4}$$

$$\boxed{R_4 = R_3 \frac{R_1}{R_2}}$$

This is the balance equation (condition) of Wheatstone bridge

The following points can be observed.

- It depends on the ratio of R_1 & R_2 hence these arms are called ratio arms.
- It works on null indication.
- The standard resistance R_3 can be varied to obtain the required balance.

Sensitivity of wheatstone bridge:-

When the bridge is balanced, the current through galvanometer is zero. But when bridge is not balanced current flows through the galvanometer causing the deflection. The amount of deflection depends on the sensitivity of the galvanometer. This sensitivity can be expressed as amount of deflection per unit current.

$$\text{Sensitivity } S = \frac{\text{Deflection } D}{\text{Current } I}$$

Another way of representing the galvanometer sensitivity is the amount of deflection per unit voltage across the galvanometer. This is called voltage sensitivity of the galvanometer. Mathematically it is denoted as.

$$S_v = \frac{\theta}{e}$$

e = Voltage across galvanometer
 θ = Deflection of galvanometer.

measured in degrees per volt (or) radians per volt.

While the bridge sensitivity is defined as the deflection of the galvanometer per unit fractional change in the unknown resistance.

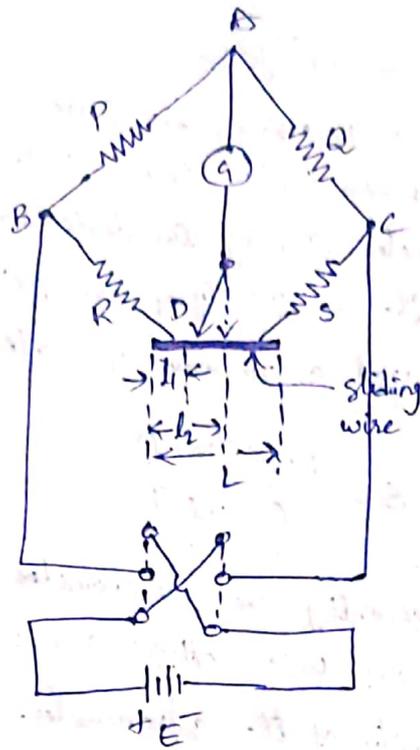
It is denoted as S_B

$$S_B = \frac{\theta}{\Delta R/R}$$

$\Delta R/R$ = unit fractional change in unknown resistance.

Carey-Foster slide wire bridge:-

A Carey-Foster slide wire bridge is the elaborated form of the wheatstone bridge. This type of bridge is most extensively used for the comparison of the two nearly equal resistances. The circuit arrangement for the Carey-Foster bridge is shown by.



The arms consisting resistances P & Q are nominal equal ratio arms. The resistance R is the resistance under test while 'S' is the standard resistance. A slide wire of length 'L' is introduced b/w the resistance R & S.

Initially a balanced condition is obtained by adjusting the sliding contact on the slide wire at a distance l_1 from the left hand side of the slide wire. After this the positions of resistances R and S are interchanged and a new balance point is obtained let the distance from the left hand of the slide wire be l_2 .

The first balance condition is given by,

$$\frac{P}{Q} = \frac{R + l_1 \gamma}{S + (L - l_1) \gamma} \quad \text{--- (1)}$$

γ = Resistance of slide wire

similarly the second balance condition is given by

$$\frac{P}{Q} = \frac{S + l_2 \gamma}{R + (L - l_2) \gamma} \quad \text{--- (2)}$$

Comparing eqⁿ (1) & (2) we can write.

$$\frac{R + l_1 \gamma}{S + (L - l_1) \gamma} = \frac{S + l_2 \gamma}{R + (L - l_2) \gamma}$$

Adding '1' on both the sides of above eqⁿ, we get

$$\frac{R + l_1 r}{S + (l - l_1)r} + 1 = \frac{S + l_2 r}{R + (l - l_2)r} + 1$$

$$\frac{R + l_1 r + S + l r - l_1 r}{S + (l - l_1)r} = \frac{S + l_2 r + R + l r - l_2 r}{R + (l - l_2)r}$$

$$\frac{R + S + l r}{S + (l - l_1)r} = \frac{S + R + l r}{R + (l - l_2)r}$$

$$R + (l - l_2)r = S + (l - l_1)r$$

$$(S - R) = (l_1 - l_2)r \quad \text{--- (3)}$$

from eqⁿ (3) it is clear that the difference b/w the standard resistance 'S' & the resistance under test 'R' is obtained as the resistance of the slide wire b/w the two balance points.

The slide wire of resistance 'r' per unit length of the slide wire can be calibrated by shunting resistances 'R' & 'S' with a known high value resistance. The effective value of the standard resistance 'S' reduces to 'S'' as complete procedure is repeated & two new balance points are obtained at lengths l_1' & l_2' from the left hand side of the slide wire.

The eqⁿ (3) can be modified as

$$(S' - R) = (l_1' - l_2')r \quad \text{--- (4)}$$

dividing eqⁿ (3) by eqⁿ (4)

$$\frac{S - R}{S' - R} = \frac{l_1 - l_2}{l_1' - l_2'}$$

$$(S - R)(l_1' - l_2') = (S' - R)(l_1 - l_2)$$

$$S(l_1' - l_2') - R(l_1' - l_2') = S'(l_1 - l_2) - R(l_1 - l_2)$$

$$S(l_1' - l_2') - S'(l_1 - l_2) = R(l_1' - l_2') - R(l_1 - l_2)$$

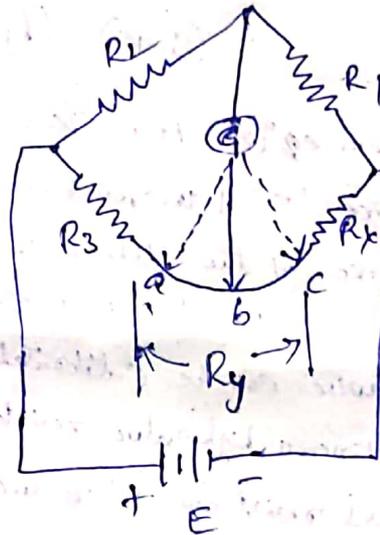
$$R = \frac{S(l_1' - l_2') - S'(l_1 - l_2)}{l_1' - l_2' - l_1 + l_2} \quad \text{--- (5)}$$

Kelvin bridge - measurement of low resistance.

In the wheatstone bridge, the bridge contact and lead resistance causes significant error, while measuring low resistances. Thus for measuring the values of resistance below 1Ω , the modified form of wheatstone bridge is used, known as kelvin bridge. Consideration of the effect of contact and lead resistances is the basic aim of the kelvin bridge.

basic circuit of kelvin bridge is shown.

The resistance R_y represents the resistance of the connecting leads from R_3 to R_x . The resistance R_x is the unknown resistance to be measured.



The galvanometer can be connected to either terminal 'a, b' or terminal 'c'.

When it is connected to 'a', the lead resistance R_y gets added to R_x hence the value measured by the bridge indicates much higher value of R_x . If the galvanometer is connected to terminal 'c', then R_y gets added to R_3 . This results in the measurement of R_x much lower than the actual value.

The point 'b' is in b/w the points 'a' & 'c', in such a way that the ratio of the resistance from 'c' to 'b' and that from 'a' to 'b' is equal to the ratio of R_1 & R_2 .

$$\frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2} \quad \text{--- (1)}$$

Now the bridge balance equation in its standard form is (1)

$$R_1 R_3 = R_2 R_x \quad (2)$$

But R_3 & R_x now are changed to $R_3 + R_{cb}$ and $R_x + R_{cb}$ respectively due to lead resistance.

$$R_1 (R_3 + R_{cb}) = R_2 (R_x + R_{cb}) \quad (3)$$

$$(R_x + R_{cb}) = \frac{R_1}{R_2} (R_3 + R_{cb}) \quad (4)$$

Now we have: $\frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2}$

$$\frac{R_{cb}}{R_{ab}} + 1 = \frac{R_1}{R_2} + 1 \quad [\text{adding 1 to both sides}]$$

$$\frac{R_{cb} + R_{ab}}{R_{ab}} = \frac{R_1 + R_2}{R_2} \quad (5)$$

But $R_{cb} + R_{ab} = R_y$ total lead resistance

Substituting in eqⁿ (5) we get

$$\frac{R_y}{R_{ab}} = \frac{R_1 + R_2}{R_2} \quad (6)$$

$$R_{cb} = \frac{R_2 R_y}{R_1 + R_2} \quad (7)$$

Now $R_{cb} + R_{ab} = R_y$

$$R_{cb} = R_y - R_{ab} \quad (8)$$

Substituting eqⁿ (7) into eqⁿ (8)

$$R_{cb} = R_y - \frac{R_2 R_y}{R_1 + R_2} = R_y \left[1 - \frac{R_2}{R_1 + R_2} \right]$$

$$R_{cb} = \frac{R_1 R_y}{R_1 + R_2}$$

Substituting these values of R_{cb} & R_{ab} in the eqⁿ (4) we get

$$R_x + \frac{R_1 R_y}{R_1 + R_2} = \frac{R_1}{R_2} \left[R_3 + \frac{R_2 R_y}{R_1 + R_2} \right]$$

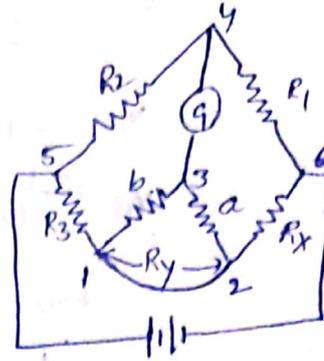
$$R_x + \frac{R_1 R_y}{R_1 + R_2} = \frac{R_1 R_3}{R_2} + \frac{R_1 R_y}{R_1 + R_2}$$

$$R_x = \frac{R_1 R_3}{R_2} + \frac{R_1 R_y}{R_1 + R_2} - \frac{R_1 R_y}{R_1 + R_2}$$

$$R_x = \frac{R_1 R_3}{R_2}$$

Kelvin's double bridge method for Low Resistance measurements:

This bridge consists of another set of ratio arms hence called double bridge circuit diagram shown below.



The second set of ratio arms is the resistances 'a' & 'b'. With the help of these resistances the galvanometer is connected to point '3'. The galvanometer gives null indication when the potential of the terminal '3' is same as the potential of the terminal '4'.

Thus $E_{45} = E_{513}$ — (1)

E_{45} = potential across R_2

E_{513} = potential across R_3 & b

The ratio of the resistances a & b is same as the ratio of R_1 & R_2

$$\frac{a}{b} = \frac{R_1}{R_2} \quad \text{--- (2)}$$

$$E_{45} = R_2 \cdot \frac{E}{R_1 + R_2} \quad \text{--- (3)}$$

Consider the path from 5-1-2-6 back to 5 through the battery E. The resistance between the terminal 1-2 is the parallel combination of R_y and $(a+b)$

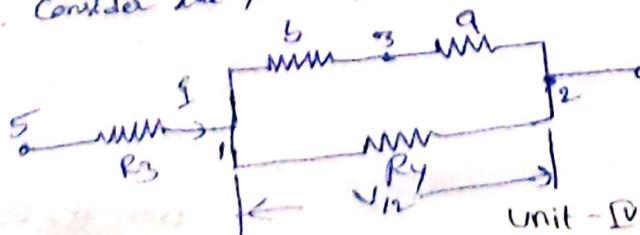
$$E = I \times [R_3 + R_y \parallel (a+b) + R_x]$$

$$E = I \left[R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} \right] \quad \text{--- (4)}$$

substituting eq (2) into equation (3)

$$E_{45} = \frac{R_2}{R_1 + R_2} \times I \left[R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} \right] \quad \text{--- (5)}$$

For E_{513} . Consider the path from the terminal 5 to 2 as shown



Now from the above figure we can write

$$V_{12} = I \left[\frac{R_y (a+b)}{R_y + a+b} \right]$$

$$V_{13} = \frac{b}{a+b} V_{12}$$

$$V_{13} = \frac{b}{a+b} \cdot I \left[\frac{R_y (a+b)}{R_y + a+b} \right] \quad \text{--- (6)}$$

$$E_{S13} = I R_3 + V_{13}$$

$$E_{S13} = I R_3 + I \cdot \frac{b}{a+b} \left[\frac{R_y (a+b)}{R_y + a+b} \right]$$

$$E_{S13} = I \left[R_3 + \frac{b}{a+b} \left[\frac{R_y (a+b)}{R_y + a+b} \right] \right] \quad \text{--- (7)}$$

Now $E_{S15} = E_{S13}$ ——— Pd balancing

$$\frac{I R_2}{R_1 + R_2} \left[R_3 + R_x + \frac{(a+b) R_y}{a+b + R_y} \right] = I \left[R_3 + \frac{b}{a+b} \left[\frac{R_y (a+b)}{a+b + R_y} \right] \right]$$

$$R_3 + R_x + \frac{(a+b) R_y}{a+b + R_y} = \left[1 + \frac{R_1}{R_2} \right] \left[R_3 + \frac{b R_y}{R_y + a+b} \right]$$

$$R_3 + R_x + \frac{(a+b) R_y}{a+b + R_y} = R_3 + \frac{R_1 R_3}{R_2} + \frac{b R_y}{R_y + a+b} + \frac{R_1 b R_y}{R_2 (R_y + a+b)}$$

$$R_x = \frac{R_1 R_3}{R_2} + \frac{b R_y}{(R_y + a+b)} \left[\frac{R_1}{R_2} - \frac{a}{b} \right]$$

$$\frac{a}{b} = \frac{R_1}{R_2} \quad ; \quad \frac{R_1}{R_2} - \frac{a}{b} = 0$$

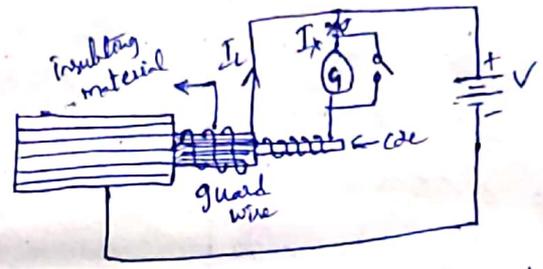
$$R_x = \frac{R_1 R_3}{R_2}$$

methods for measurement of high resistances:-

- 1) Direct deflection method.
- 2) Loss of charge method
- 3) megohm-bridge
- 4) megger.

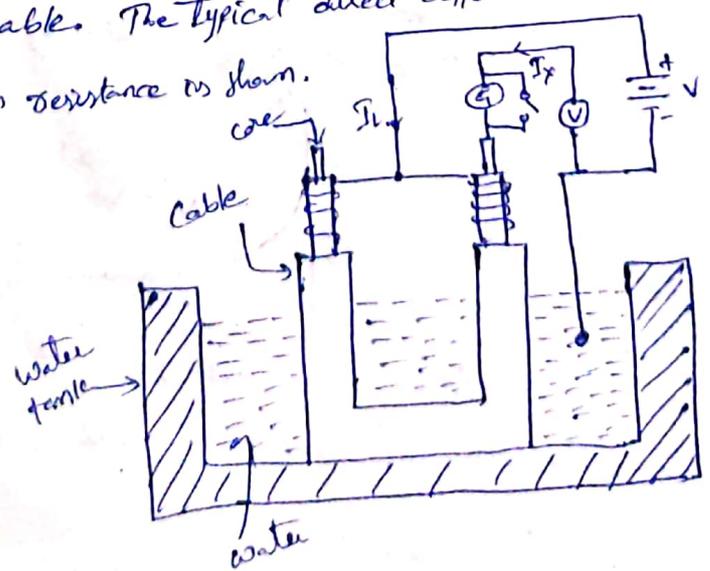
i) Direct deflection method:-

In this method, a high resistance ($> 1000 \Omega$) and very sensitive moving coil galvanometer is connected in series with the resistance to be measured along with supply voltage.



By this method insulation resistance can be obtained with respect to the deflection of galvanometer.

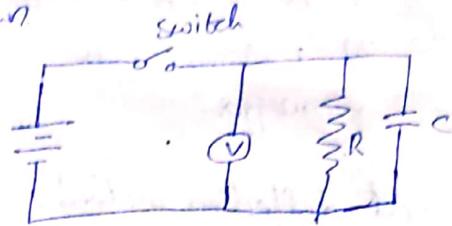
The galvanometer measures current b/w Coe & metal sheath. The leakage currents over the surface of the insulating material are carried by the guard wire wound on the insulation. These currents do not flow through galvanometer. The ratio of the voltage applied b/w Coe & metal sheath & current flowing b/w them gives the insulation resistance of the cable. The typical direct deflection method for measuring the insulation resistance is shown.



ii) Loss of charge method:-

This is very typical method of measuring the insulation resistance of very high value. The typical arrangement of resistance measurement is shown

The resistance to be measured is shunted by a known value



Capacitor. The voltage across parallel combination is measured using electrostatic voltmeter. The circuit is driven by a D.C voltage source of value V . This voltage is applied to the circuit through a switch.

Initially switch is kept open. When the switch is closed at certain instant, the capacitor 'C' starts charging.

The voltage across 'C' is given by

$$V_c = V \left[1 - e^{-\frac{t}{RC}} \right]$$

Then at certain instant say $t = t_1$, switch is opened.

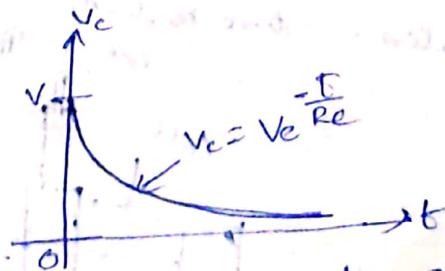
Then capacitor 'C' starts discharging through R. Then at instant voltage is given by

$$V_c = V e^{-t/RC}$$

$$\frac{V_c}{V} = e^{-\frac{t}{RC}}$$

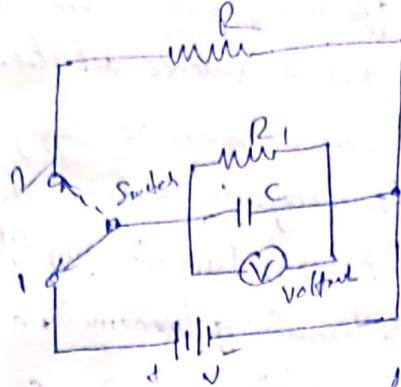
Simplifying $R = \frac{t}{C \ln \frac{V}{V_c}} = \frac{0.4343 t}{C \log_{10} \frac{V}{V_c}}$

The variation of voltage across C is given by



If the value of resistance R is very large, then capacitor C requires more time for discharging. In such cases, the process becomes time consuming.

This method can be used effectively for measurement of high resistances, but it needs a capacitor with a high leakage resistance. The typical circuit arrangement is shown.



The circuit consists high insulation resistance R to be measured along with capacitor of known value C shunted with electrostatic voltmeter and leakage resistance R_1 .

Initially, capacitor C is charged to suitable voltage say V_1 by moving switch to position 1. Then switch is moved to position 2. The capacitor starts discharging through parallel combination of R and R_1 . At certain instant t voltage across capacitor drops down from V_1 to V_2 . Let the equivalent resistance through which C discharges be denoted by R' where $R \parallel R_2$.

The expression for current any instant t is given by

$$i = -\frac{dq}{dt} = -C \frac{dV}{dt} \quad \text{--- (1)}$$

$$i = \frac{\text{Potential drop across } R'}{R'} = \frac{V}{R'} \quad \text{--- (2)}$$

Comparing eqⁿ (1) & (2) we can write

$$\frac{V}{R'} = -C \frac{dV}{dt} \quad \text{or} \quad \frac{dV}{V} = -\frac{dt}{RC}$$

integrating both sides

$$[\ln V]_{V_1}^{V_2} = \left[\frac{-t}{RC} \right]_0^t$$

$$\ln \frac{V_2}{V_1} = \frac{-t}{RC}$$

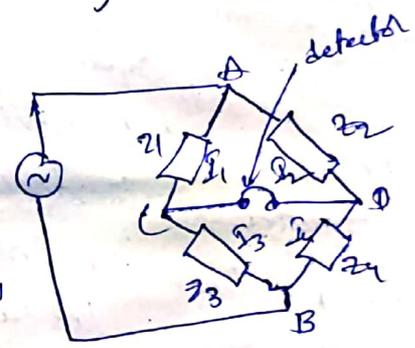
$$V_2 = V_1 e^{-\frac{t}{RC}}$$

$$V_2 = V_1 e^{-\frac{t}{R_1 C}}$$

(3)

AC bridges:— An A.C bridge in its basic form consists of four arms, a source of excitation and a balance detector. Each arm consists of an impedance. The source is an ac supply which supplies AC voltage at the required frequency. The balance detectors commonly used for A.C bridges are headphones, tunable amplifier circuits, & vibration galvanometers. The vibration galvanometers are useful for low audio frequency range from 5 Hz to 1000 Hz. The simple a.c bridge is the outcome of the wheatstone bridge. The impedances at audio and radio frequency ~~at~~ ~~and~~ range can be easily determined by such simple ac wheatstone bridge.

For the balance bridge, the potential at point C must be same as the potential of point D. Hence potential must be equal in terms of amplitude as well as phase.



The drop from A to C must be equal to drop across A to D in both magnitude and phase if the bridge balance.

$$E_{AC} = E_{AD}$$

$$I_1 Z_1 = I_2 Z_2 \quad \text{--- (1)}$$

When the bridge is balanced, no current flows through the headphones.

$$I_3 = I_1 \quad \& \quad I_4 = I_2$$

$$I_1 = \frac{E}{Z_1 + Z_3} \quad \text{--- (2)}, \quad I_2 = \frac{E}{Z_2 + Z_4} \quad \text{--- (3)}$$

Substituting eqⁿ (2) & (3) into eqⁿ (1), we get

$$\frac{E \cdot Z_1}{Z_1 + Z_3} = \frac{E \cdot Z_2}{Z_2 + Z_4}$$

$$Z_1 Z_2 + Z_1 Z_4 = Z_2 Z_3 + Z_2 Z_4$$

$$Z_1 Z_4 = Z_2 Z_3 \quad \text{--- (4)}$$

The eqⁿ ⑤ is the balance equation in the Impedance form.

In the admittance form the condition can be expressed as

$$Y_1 Y_4 = Y_2 Y_3$$

The admittance is the reciprocal of the impedance, and in the polar form the impedances are expressed as.

$$Z_1 = z_1 \angle \theta_1$$

$$Z_2 = z_2 \angle \theta_2$$

$$Z_3 = z_3 \angle \theta_3$$

$$Z_4 = z_4 \angle \theta_4$$

z_1, z_2, z_3, z_4 are the magnitudes and $\theta_1, \theta_2, \theta_3, \theta_4$ are the phase angles.

Note that the product of the impedances must be carried out in polar form where magnitudes get multiplied & phase angles get added.

Substituting in eqⁿ ⑤ we get

$$z_1 \angle \theta_1 \times z_4 \angle \theta_4 = z_2 \angle \theta_2 \times z_3 \angle \theta_3$$

$$z_1 z_4 \angle \theta_1 + \theta_4 = z_2 z_3 \angle \theta_2 + \theta_3$$

Equating magnitudes both sides we get

$$z_1 z_4 = z_2 z_3$$

Equating phase angles we get

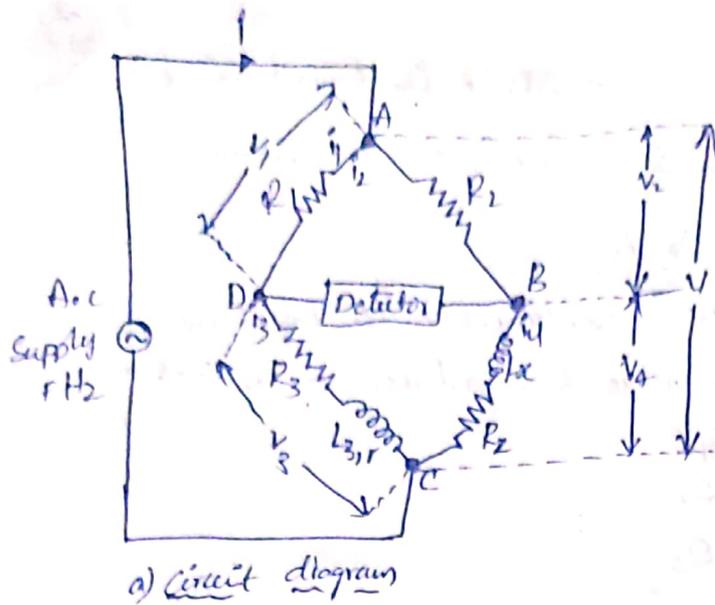
$$\theta_1 + \theta_4 = \theta_2 + \theta_3$$

7.6 :- Maxwell's Bridge

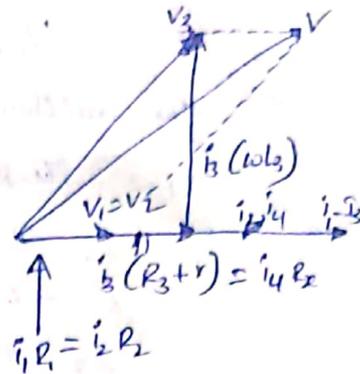
Maxwell's bridge can be used to measure inductance by comparison either with a variable standard self inductance or with a standard variable capacitance. These two measurements can be done by using the Maxwell's Bridge in two different forms

7.6.1 :- Maxwell's Inductance Bridge

Using this bridge we can measure inductance by comparing it with a standard variable self inductance arranged in bridge circuit as shown in fig. 7.7 (a)



a) Circuit diagram



b) Phasor diagram

Maxwell's inductance Bridge

Consider Maxwell's inductance bridge as shown in the Fig 2.2 Two branches consist of non-inductive resistance R_1 and R_2 . One of the arms consists variable inductance with series resistance r . The remaining arm consist unknown inductance L_x .

At balance, we get condition as

$$\frac{R_1}{(R_3+r) + j\omega L_3} = \frac{R_2}{R_x + j\omega L_x} \quad (1)$$

$$R_1(R_x + j\omega L_x) = R_2[(R_3+r) + j\omega L_3]$$

$$R_1 R_x + j\omega R_1 L_x = R_2(R_3+r) + j\omega R_2 L_3$$

Equating imaginary terms, we can write

$$R_1 L_x = R_2 L_3$$

$$L_x = \frac{R_2}{R_1} L_3 \quad (2)$$

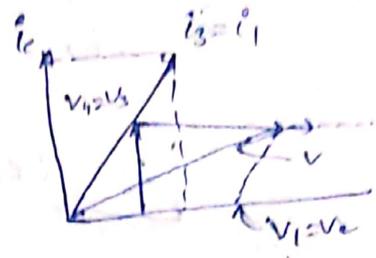
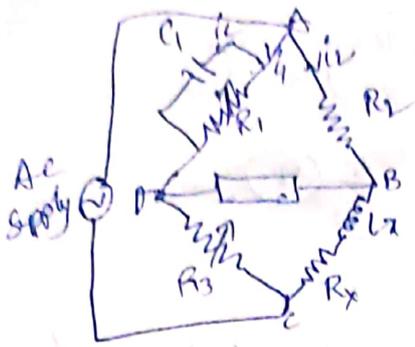
Equating real terms we can write

$$R_1 R_x = R_2(R_3+r)$$

$$R_x = \frac{R_2}{R_1} (R_3+r) \quad (3)$$

(ii) maxwell's inductance capacitance bridge

using this bridge we can measure inductance by comparing with a variable standard capacitor.



one of the ratio arms consists of resistance and capacitance in parallel. hence it is simple to write the bridge equations in the admittance form

The bridge balance equation is

$$Z_1 Z_x = Z_2 Z_3 \quad \text{--- (1)}$$

$$Z_x = \frac{Z_2 Z_3}{Z_1} = Z_2 Z_3 Y_1$$

$$Y_1 = \frac{1}{Z_1} \quad \text{i.e. } R_1 \text{ in parallel with } C_1$$

$$Z_2 = R_2$$

$$Z_3 = R_3$$

$$Z_x = R_x + j\omega L_x, \text{ as } L_x \text{ in series with } R_x.$$

Now $Y_1 = \frac{1}{R_1} + j\omega C_1$ --- (2)

$$Z_1 = R_1 || j \left(\frac{1}{\omega C_1} \right) \quad \text{as } \frac{1}{j} = -j$$

Substituting all the values in eqn (1) we get

$$R_x + j\omega L_x = R_2 R_3 \left[\frac{1}{R_1} + j\omega C_1 \right]$$

$$R_x + j\omega L_x = \frac{R_2 R_3}{R_1} + j R_2 R_3 \omega C_1 \quad \text{--- (3)}$$

equating real parts.

$$R_x = \frac{R_2 R_3}{R_1} \quad \text{--- (4)}$$

equating imaginary parts

$$\omega L_x = R_2 R_3 \omega C_1$$

$$L_x = R_2 R_3 C_1 \quad \text{--- (5)}$$

The resistances are expressed in ohms, inductance in henries
capacitance in farads.

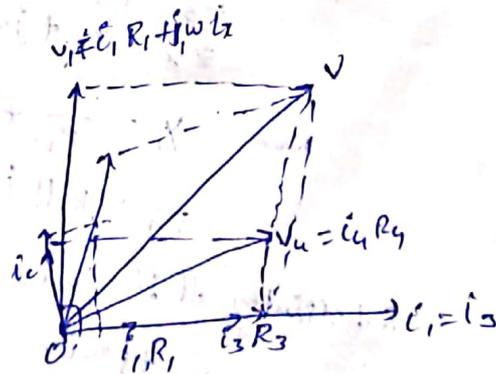
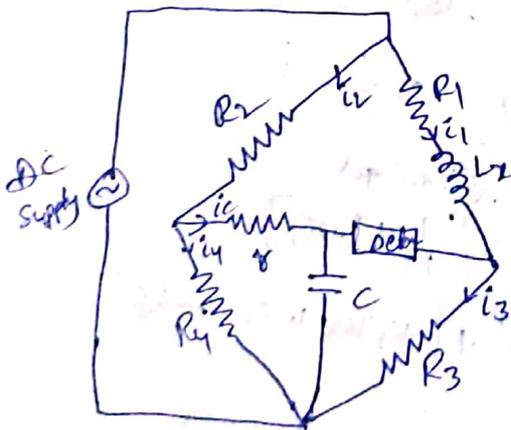
The quality factor of the coil is given by

$$Q = \frac{\omega L_x}{R_x} = \frac{\omega R_2 R_3 C_1}{\left(\frac{R_2 R_3}{R_1}\right)}$$

$$Q = \omega R_1 C_1 \quad \text{--- (7)}$$

Anderson bridge:— It is another important A.C bridge used for the measurement of self inductance in terms of a standard capacitor. Actually this bridge is nothing but Maxwell's bridge in which also the value of self inductance is obtained by comparing it with a standard capacitor.

This bridge is basically used for the precise measurement of inductance over a wide range of values. The Anderson bridge is shown



one arm of the bridge consists of unknown inductor L_x with known resistance in series with L_x . This resistance R_1 includes resistance of the inductor. C is the standard capacitor with r , R_2 , R_3 and R_4 are non-inductive known resistances.

The bridge balance equations are

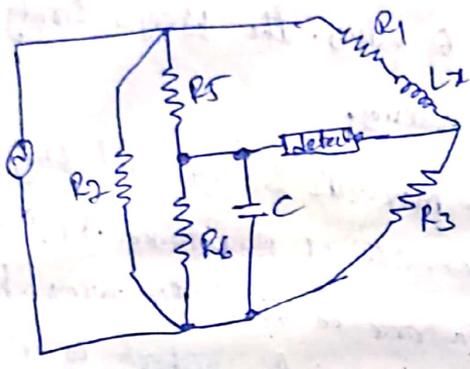
$$i_1 = i_3, i_2 = i_4 + i_c; V_2 = i_2 R_2, V_3 = i_3 R_3$$

$$V_1 = V_2 + i_c \gamma \text{ and } V_4 = V_3 + i_c \gamma, V_1 = i_1 R_1 + i_1 \omega L_1$$

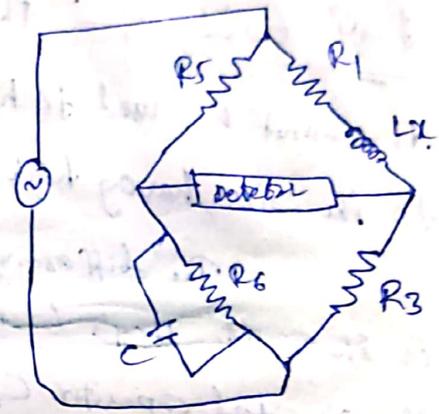
$$V_4 = i_4 R_4$$

$$V = V_2 + V_4 = V_1 + V_3$$

To find balance eqⁿ transforming a star formed by R_2, R_4 and γ into its equivalent delta as shown.



(a)



(b)

The elements in equivalent delta are given by

$$R_5 = \frac{R_2 \gamma + R_4 \gamma + R_2 R_4}{R_4}$$

$$R_6 = \frac{R_2 \gamma + R_4 \gamma + R_2 R_4}{R_2}$$

$$R_7 = \frac{R_2 \gamma + R_4 \gamma + R_2 R_4}{\gamma}$$

Now R_7 shunts the source, hence it does not affect the balance conditions. Thus by neglecting R_7 and rearranging a network we get a Maxwell inductance bridge.

The balance equations are given by

$$L_x = C R_3 R_5$$

$$R_1 = R_3 \frac{R_5}{R_6}$$

Substituting values of R_5 & R_6 , we can write

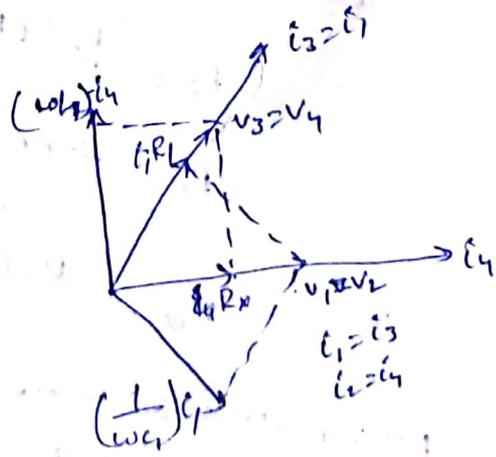
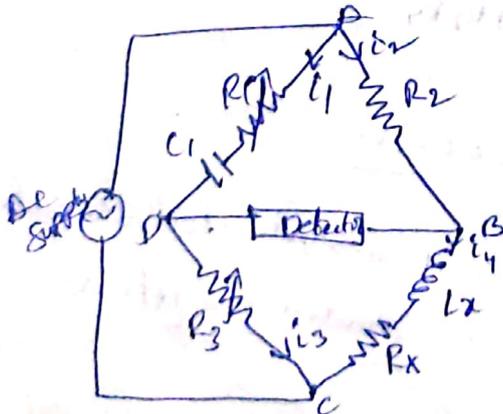
$$L_x = \frac{CR_3}{R_4} [R_2 R_4 + R_4 R_7 + R_2 R_4]$$

$$R_1 = \frac{R_2 R_3}{R_4}$$

Hay's bridge - The limitation of Maxwell's bridge is that it cannot be used for high Q values. The Hay's bridge is suitable for the coils having high Q values.

The difference in Maxwell's bridge and Hay's bridge is that the Hay's bridge consists of Resistance R_1 in series with the standard capacitor C_1 in one of the ratio arms. Hence for larger phase angles R_1 needed is very low, which is practicable. Hence bridge can be used for the coils with high Q values. The Hay's bridge is shown.

AC
SQR



(11)
The various constants of the bridge are:

$$Z_1 = R_1 - jX_{C_1} = R_1 - j\left(\frac{1}{\omega C_1}\right)$$

$$Z_2 = R_2 \quad \& \quad Z_3 = R_3$$

$$Z_4 = Z_x = R_x + j(\omega L_x)$$

At the balance condition.

$$\overline{Z_1 Z_x} = \overline{Z_2 Z_3}$$

$$\left[R_1 - j\left(\frac{1}{\omega C_1}\right)\right](R_x + j(\omega L_x)) = R_2 R_3$$

$$R_1 R_x - j\left(\frac{R_x}{\omega C_1}\right) + j\omega R_1 L_x + \frac{L_x}{C_1} = R_2 R_3$$

$$\left[R_x R_1 + \frac{L_x}{C_1}\right] + j\left[\omega R_1 L_x - \frac{R_x}{\omega C_1}\right] = R_2 R_3 \quad \text{--- (1)}$$

Equating the real parts of both sides.

$$R_x R_1 + \frac{L_x}{C_1} = R_2 R_3 \quad \text{--- (2)}$$

Equating the imaginary parts of both sides of eqⁿ (1)

$$\omega R_1 L_x - \frac{R_x}{\omega C_1} = 0 \quad \text{--- (3)}$$

To obtain R_x and L_x ; solve eqⁿ (2) & eqⁿ (3) simultaneously from eqⁿ (3).

$$\omega R_1 L_x = \frac{R_x}{\omega C_1}$$

$$L_x = \frac{R_x}{\omega^2 R_1 C_1} \quad \text{--- (4)}$$

Substituting in eqⁿ (2)

$$R_x R_1 + \frac{R_x}{\omega^2 R_1 C_1} = R_2 R_3$$

$$R_x \left[R_1 + \frac{1}{\omega^2 R_1 C_1} \right] = R_2 R_3$$

$$R_x \left[\frac{1 + \omega^2 R_1^2 C_1^2}{\omega^2 R_1 C_1} \right] = R_2 R_3$$

$$R_x = \frac{\omega^2 R_1 C_1^2 R_2 R_3}{1 + \omega^2 R_1^2 C_1^2} \quad \text{--- (5)}$$

Substituting eqⁿ (5) in eqⁿ (4) we get

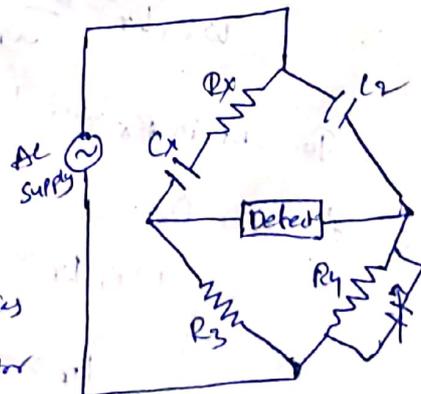
$$L_x = \frac{\omega^2 R_1 C_1^2 R_2 R_3}{(1 + \omega^2 R_1^2 C_1^2) \omega^2 R_1 C_1}$$

$$L_x = \frac{R_2 R_3 C_1}{1 + \omega^2 R_1^2 C_1^2} \quad \text{--- (6)}$$

using eqⁿ (5) & eqⁿ (6) the unknown resistance and inductance can be calculated.

Schering bridge:- It is one of the most widely used ac bridges for the measurement of unknown capacitors; dielectric loss and power factor.

The diagram shows the connections of Schering bridge. It can be used for low voltages. The C_x is perfect capacitor to be measured. R_x is series resistance. C_2 is standard air capacitor having very stable value. R_3 & R_4 are non-inductive resistances while C_4 is variable capacitor.



From the general balance equation.

$$Z_1 Z_4 = Z_2 Z_3$$

$$Z_1 = R_x - j \frac{1}{\omega C_x}$$

$$Z_2 = \frac{-j}{\omega C_2}$$

$$Z_3 = R_3$$

$$Z_4 = R_4 \parallel \frac{-j}{\omega C_4} = \frac{R_4 \left(\frac{-j}{\omega C_4} \right)}{\left(R_4 - j \frac{1}{\omega C_4} \right)}$$

$$Z_4 = \frac{-j R_4}{\omega R_4 C_4 - j} = \frac{-j R_4 (\omega R_4 C_4 + j)}{(\omega R_4 C_4 - j)(\omega R_4 C_4 + j)} = \frac{R_4 - j \omega R_4^2 C_4}{\omega^2 R_4^2 C_4^2 + 1}$$

$$Z_1 = \frac{Z_2 Z_3}{Z_4} = \frac{\left(\frac{-j}{\omega C_2} \right) (R_3)}{\left(\frac{R_4 - j \omega R_4^2 C_4}{1 + \omega^2 R_4^2 C_4^2} \right)} = \frac{(1 + \omega^2 R_4^2 C_4^2) R_3 \left(\frac{-j}{\omega C_2} \right)}{R_4 - j \omega R_4^2 C_4}$$

Rationalising, $Z_1 = R_3 (1 + \omega^2 R_4^2 C_4^2) \left\{ \frac{-j (\omega R_4 + j \omega R_4^2 C_4)}{R_4 + \omega^2 R_4^2 C_4^2} \right\}$

$$R_x - j \frac{1}{\omega C_x} = \frac{R_3 (1 + \omega^2 R_4^2 C_4^2)}{R_4 (1 + \omega^2 R_4^2 C_4^2)} \left\{ \frac{R_4 C_4}{C_2} - \frac{j R_4}{\omega C_2} \right\}$$

equating real and imaginary parts,

$$R_x = \frac{R_3}{R_4} \times \frac{R_4 C_4}{C_2} = \frac{R_3 C_4}{C_2}$$

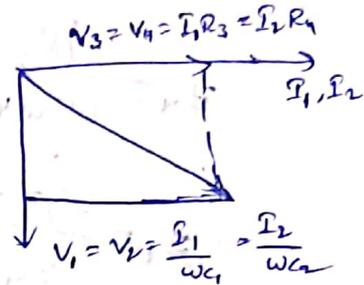
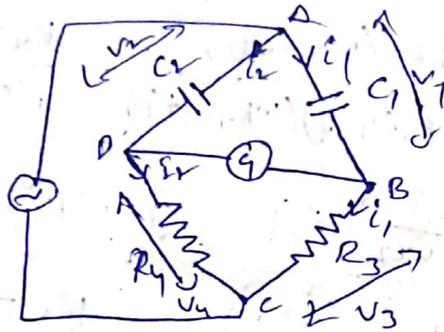
$$-j \frac{1}{\omega C_x} = -j \frac{R_3}{R_4} \times \frac{R_4}{\omega C_2} = -j \left[\frac{1}{\frac{R_4}{R_3} \omega C_2} \right]$$

$$\omega C_x = \frac{R_4}{R_3} \omega C_2$$

$$C_x = \frac{R_4}{R_3} C_2$$

De Sauty bridge :-

The basic application of the De Sauty bridge is to compare two capacitance. The circuit diagram of De Sauty bridge is shown



The De Sauty bridge consists a capacitor C_1 which is capacitance under test in branch AB. The branch AD consists a known, standard capacitor. The remaining branches BC & CD consists non-inductive resistors R_3 & R_4 respectively.

At balance, we get condition \rightarrow

$$\frac{-j \times C_1}{R_3} = \frac{-j \times C_2}{R_4}$$

$$\left(\frac{-j}{\omega C_1}\right) R_4 = \left(\frac{-j}{\omega C_2}\right) R_3$$

$$C_1 = \frac{C_2 R_4}{R_3}$$

The balance in the bridge can be achieved by varying either R_3 & R_4 . Under the balanced condition the vector diagram for the De Sauty bridge is shown.

The main advantage of De Sauty bridge is that it is very simple in construction and the measurement of the capacitor is done with simplicity.